Adults should be able to recognize relationship and apply whatever tools necessary to derive appropriate conclusions from the data based on the function generated, whether it is quadratic, linear, or exponential (6)

**Big Ideas**

* Algebraic reasoning is a process of describing and analyzing (e.g., predicting) generalized mathematical relationships and change using words and symbols
* Comparing mathematical relationships helps us see that there are classes of relationships and provides insight into each member of the class.
* Different representations of relationships (e.g., numeric, graphic, geometric, algebraic, verbal, concrete/pictorial) highlight different characteristics or behaviours, and can serve different purposes.
* Limited information about a mathematical relationship can sometimes, but not always, allow us to predict other information about that relationship.

**Big Ideas for Quadratics**

* The graphical, algebraic, numerical, geometric or verbal representation of a quadratic function reveals different information about the concrete/pictorial representation of the quadratic.
* Consideration of the geometric representation allows understanding of the predictable affect of parameters on the function. (something about transformations and combining functions)

**Principals/Characteristics**

* The nature of the quantities in a relationship determines what values of the input and output quantities are reasonable. (1)
* The desired answer determines the reasonable model(s) that can be used and the ‘path’ that can be taken. (e.g., looking for height at t=3, don’t complete the square/factor) (new)

**Pedagogical Ideas**

* Technology can enhance student understanding of the advantages of different representations of functions and provide insight to the classes of functions and their defining shapes (6)

Student Misconstructs

From 7

Students may

* View a graph as a sketch through three or four points and never consider trends in families of functions
* Not recognize the role or significance of the x-intercepts
* Rarely consider ways in which the graph provides visual insight into the behaviour of the function
* Fail to understand the power of the graphical representation
* Fail to connect the graphical representation to other representations
* Use symbols before they understand the meanings behind the symbols

Teacher should

* Engage students in connecting various representations of a function
* Have students solve problems graphically before algebraically in order to build better connections between the graph and the equation (not true for algebraic to graphic)
* Have students build quadratic functions by multiplying linear functions (rather than seeing them as a new, unrelated topic)

From 6

Students may

* Not understand the relevance of a solution to an equation
* Not comprehend the beauty and significance of the graph (because there is an overdependence on the procedure of graphing)

Teachers should

* Expose students to quadratic equations that cannot be factored in order to expand their understanding that factoring is not always the appropriate strategy
* Engage students in judging reasonableness of the answer and appropriate types of solutions, which requires students to understand the number system

From 2

Sfard’s developmental phases:

* Interiorization: the learner is capable of dealing with operational processes on symols
  + E.g., the student uses the idea of variables in order to manipulate a formula and find values
* Condensation: the student has developed the ability to use mapping as a whole, without looking into its specific values
  + Eventually, the student can investigate functions, draw their graphs and combine couples of functions
* Reification: the student is able to handle functions as objects
  + E.g., when the ‘unknowns’ or ‘variables’ are functions and the student has the ability to talk about general properties of different processes performed on functions as integration processes

Students may

* Have two simultaneous evoked images for one concept, which results in cognitive conflict
  + E.g., quadratic function evokes quadratic formula and parabola
* Lack understanding about when and how to use the vertex (h,k) with the vertex form, y=x(a-h)2+k, which is demonstrated through students using the same process (algebraic to graphical) regardless of the task
* Lack understanding of why a parabola horizontally or vertically shifts
* The phenomenon of compartmentalization – having two different, potentially conflicting algebraic and graphical schemes in the cognitive structure
* Have a tendency to use standard form over vertex form
* Misinterpreting the point (b,c) as the vertex of the standard form y=ax2+bx+c

Teachers should

* Use a well-designed set of tasks or examples that emphasize the distinction between the quadratic formula and quadratic function
* Present the vertex form by connecting it to the standard form and emphasize the connections to the graph as well as information about how and when to use it
* Be open and prepared to communicate with students in terms of confronting, discussing, and dealing with these conflicting schemes
* Avoid sequential presentation of the forms and instead simultaneously introduce vertex and standard form, emphasize the differences between them and reveal the underlying thoughts that generated vertex form
* Connecting the problem, the graph and the vertex to avoid isolated and unconnected knowledge in the student’s cognitive structure
* Interrelate graphical and algebraic representations
* Monitor student thinking to expose misconstructs and to allow deeper connections to be established

From 3

Teachers should

* Focus on quantitative relationships that can be represented by both linear and quadratic functions
* Examine the ways in which quadratic growth differs from linear growth

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